

## Some of the features of the viscoplastic media

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### Abstract

The article considers the influence of the structure and the chemical composition of ground blast furnace slag on rheological properties of slag suspensions. Different composition and structure of blast furnace slags were studied. The features of structural and mechanical properties of suspensions are revealed at shear rate  $1\div 50\text{ c}^{-1}$ . The theoretical model of the flow in a flat endless channel at a final external pressure difference is presented on the basis of experimental data. The exact solution of the system of equations describing the considered flow is obtained.

**Keywords:** blast furnace slag, structure, chemical composition, rheological properties, quasi Bingham medium

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## INTRODUCTION

Production processes at high temperatures such as metallurgical process are accompanied by the release of by-products called slag. Blast furnace slag of the metallurgical slag is particularly attracted the attention of builders because they are characterized by a constant chemical composition and environmental safety. Blast furnace slags also have astringent properties which is very important for the construction works since ancient times. As binders blast furnace slag are used in construction works all over the world in particular in Russia (Danilovich and Skanavi 1988, Gergichny 2013). They are widely used for the manufacture of plaster and masonry mortars, various concretes and production of construction and heat-resistant materials for various purposes. For example, blast-furnace granulated slag is the main component in the production of cements.

When mixing with slag water blast furnace slags form a visco-plastic suspensions that harden after setting under certain conditions. Rheological features (Altoubat et al. 2016, Mo et al. 2015) of these suspensions affect the structural and mechanical properties of the final product used in construction. In this regard, in this work the task is set to investigate rheological features of slag suspensions by examples of specific blast furnace slag produced by Novokuznetsk, Mariupol and Chelyabinsk metallurgical plants.

Thus the purpose of this work is an experimental study of rheological features of slag suspensions by slag samples taken from Novokuznetsk, Mariupol and Chelyabinsk metallurgical plants and theoretical description of slag suspension flow on the basis a new

rheological model for plate endless canal at a final external pressure difference.

**Technique.** The rotary viscometer "Rheotest-2" having coaxial cylinders (Mal'kova 2004) was used for experimental researches of the above slag suspensions. The rheograms showing the change of shear stress  $\tau$  on the shear rate change  $\dot{\gamma}$  are obtained. The change interval of the shear rate was  $1\div 50\text{ c}^{-1}$  and shear stress varied in the range of  $120\div 250\text{ Pa}$ . **Figs. 1-3** show the results of experimental studies of slag suspensions without any additives.

**Main part.** The given programs (see **Figs. 1-3**) show that the suspensions of the Novokuznetsk and Mariupol samples have very small yield strength not more than  $10\div 15\text{ Pa}$ . For them, the value of the dynamic viscosity is, respectively, 47 and 35 mPa·c. The Chelyabinsk suspension sample has a yield strength of 35 Pa and its the dynamic viscosity is  $120\div 140\text{ mPa}\cdot\text{c}$ .

From the above rheograms it can be seen that the dependence of the shear stress on the flow velocity gradient does not correspond to the dependence expected for Bingham media (Gnoevoj et al. 2001a, 2001b, Vishnyakov and Pokrovskij 2013).

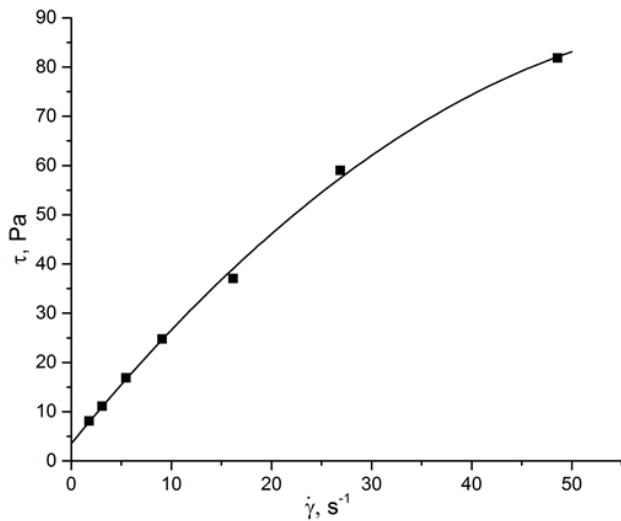
## RHEOLOGICAL MODEL

The course of curves in the above reogramme detect weak non-linearity of the shear rate on shear rate  $\dot{\gamma}$  in comparison with the dependencies typical for Bingham

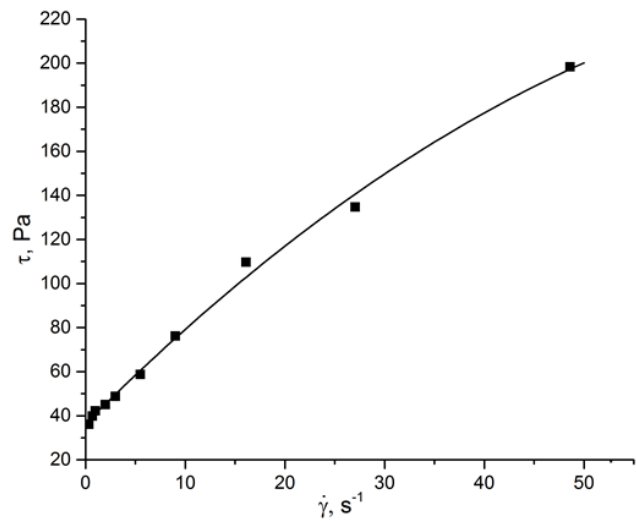
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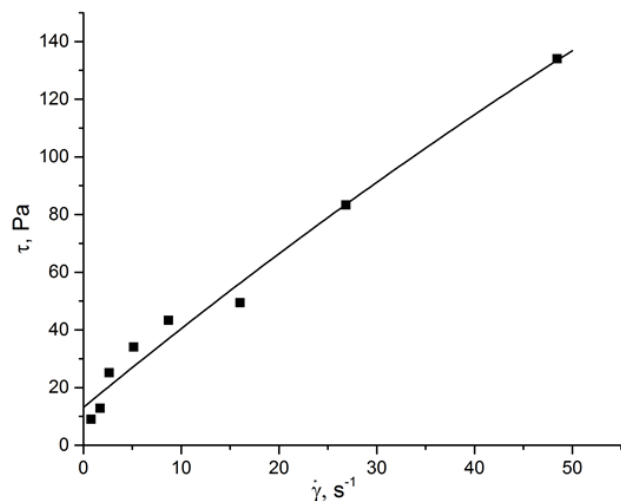
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**Fig. 1.** Rheogram of shear stress on shear rate of Novokuznetsk aphanitic slag suspension. Experimental points are marked by the symbol  $\blacksquare$ . The solid curve corresponds to the theoretical dependence  $\tau = 3.50754 + 2.48875\dot{\gamma} - 0.01793(\dot{\gamma})^2$



**Fig. 3.** Rheogram of shear stress on shear rate of Chelyabinsk vitrophyric acid slag suspension. Experimental points are marked by the symbol  $\blacksquare$ . The solid curve corresponds to the theoretical dependence  $\tau = 36.39485 + 4.53185\dot{\gamma} - 0.02514(\dot{\gamma})^2$



**Fig. 2.** Rheogram of shear stress on shear rate of Mariupol vitrophyric hydroxidic slag suspension. Experimental points are marked by the symbol  $\blacksquare$ . The solid curve corresponds to the theoretical dependence  $\tau = 13.15087 + 2.79153\dot{\gamma} - 0.00636(\dot{\gamma})^2$

media (Mo et al. 2015). In this regard, we assume the rheological equation that describes the above suspensions is as follows

$$\tau = \tau_0 + \mu(\dot{\gamma}) \cdot \dot{\gamma} \tag{1}$$

where dynamic viscosity  $\mu(\dot{\gamma})$  is a function of  $\dot{\gamma}$  and the dependence of  $\mu(\dot{\gamma})$  is determined by the equality:

$$\mu(\dot{\gamma}) = \mu - \lambda \dot{\gamma} \tag{2}$$

where  $\mu$  is an usual dynamic viscosity which is a constant and the coefficient  $\lambda$  characterizes the consistency of the suspension.

By substituting Eq. (2) into Eq. (1), we transform the rheological equation to the form:

$$\tau = \tau_0 + \mu \cdot \dot{\gamma} - \lambda \cdot (\dot{\gamma})^2 \tag{3}$$

In connection with the presence in Eq. (3) nonlinear term, we call the considered slag suspensions described by Eq. (3) as quasi Bingham media.

By analogy with the work (Mo et al. 2015), we now write a generalized rheological equation of quasi Bingham media corresponding to Eq. (3) :

$$T = \tau_0 + 2\mu \cdot H - \lambda H^2 \tag{4}$$

where

$$T = \left\{ \frac{1}{6} [(\tau_{xx} - \tau_{yy})^2 + (\tau_{yy} - \tau_{zz})^2 + (\tau_{zz} - \tau_{xx})^2] + \tau_{xy}^2 + \tau_{xz}^2 + \tau_{yx}^2 \right\}^{1/2} \tag{5}$$

$$H = \left\{ \frac{1}{6} [(\varepsilon_{xx} - \varepsilon_{yy})^2 + (\varepsilon_{yy} - \varepsilon_{zz})^2 + (\varepsilon_{zz} - \varepsilon_{xx})^2] + \varepsilon_{xy}^2 + \varepsilon_{xz}^2 + \varepsilon_{yx}^2 \right\}^{1/2} \tag{6}$$

$T$  is the magnitude of the tangential stresses,  $H$  is the magnitude of shear deformation rates in any point of medium.

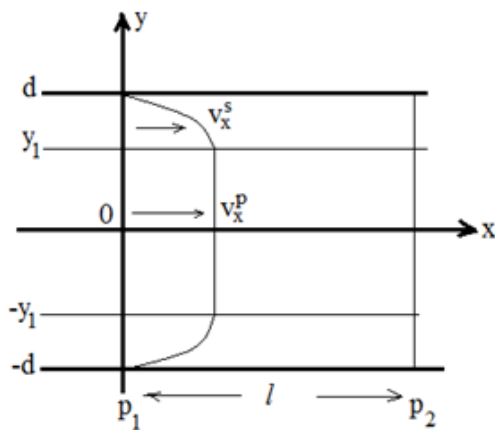
The full system of equations of the quasi-Bingham medium flow is analogous to the full system of equations of Bingham medium of the work [4] except that Eq. (4) is now used instead of the rheological equation

$$T = \tau_0 + 2\mu \cdot H$$

represented in (Gnoevoj et al. 2004).

### STATIONARY FLOW OF A QUASI BINGHAM MEDIUM BETWEEN PARALLEL PLANES

Consider the solution of the problem of steady flow of incompressible quasi-Bingham medium in a flat channel with a rheological law Eq. (4) at constant temperature if the pressure difference is set by analogy



**Fig. 4.** Scheme of quasi-Bingham medium flow between parallel planes.  $\pm d$  – channel boundaries,  $\pm y_1$  – boundaries between shear (s) and plastic (p) flow regions,  $v_x$  – flow velocity

with work (Mo et al. 2015). In **Fig. 4**, we present a scheme of such a quasi-Bingham medium flow.

This flow is symmetric with respect to the plane  $y = 0$ . We formulate the boundary values analogously to the boundary conditions represented in the work [4]. For velocities:

$$v_x \equiv v = 0 \text{ at } y = \pm d, v^s = v^p \text{ at } y = \pm y_1;$$

for pressures:

$$p = p_1 \text{ at } x = 0, p = p_2 \text{ at } x = l, p_1 > p_2.$$

Due to the steady state of the flow

$$\frac{dv}{dt} = 0,$$

and due to the incompressibility of the medium, the velocity function is function only a variable  $y$ :  $v = v(y)$ . In addition, there is no movement of the medium along the OY axis:  $v_y = 0$ .

We write down a system of equations for a given plane flow of a quasi Bingham medium using Eqs (4) – (6) analogously to the work (Mo et al. 2015):

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \quad \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} = 0 \quad (7)$$

As  $v = v(y)$  then

$$\varepsilon_{xx} = \frac{\partial v}{\partial x} = 0.$$

It is also clear that  $\varepsilon_{zz} = \frac{\partial v_z}{\partial z} = 0$ . Then it is follows from the incompressibility condition of the medium  $\varepsilon_{yy} = \frac{\partial v_y}{\partial y} = 0$ . Besides  $\tau_{zy} = \varepsilon_{zy} = 0, \tau_{xz} = \varepsilon_{xz} = 0$  and  $\varepsilon_{xy} = \frac{1}{2} \frac{dv}{dy}$ . Therefore, from Eqs (6), (5) we have

$$H = \varepsilon_{xy} = \frac{1}{2} \frac{dv}{dy}$$

and

$$T = \frac{1}{2} \sqrt{(\tau_{xx} - \tau_{yy})^2 + 4\tau_{xy}^2}$$

Due to the symmetry of the problem mentioned above, the rheological equation Eq. (4) takes the form at  $y \geq 0$ :

$$\frac{1}{2} \sqrt{(\tau_{xx} - \tau_{yy})^2 + 4\tau_{xy}^2} = -\tau_0 + \mu \frac{dv}{dy} - \lambda \left(\frac{dv}{dy}\right)^2 \quad (8)$$

In accordance with (Gnoevoj et al. 2004), the equation

$$\frac{dv}{dy} (\tau_{xx} - \tau_{yy}) = 0 \quad (9)$$

closes the system of equations for this description.

Thus it is necessary to solve the system of Eqs (7) – (9) taking into account the above boundary conditions to describe the stationary flow of the quasi Bingham medium enclosed between two planes.

We will look for the solution of the problem in the upper half-plane of the **Fig. 4**:  $0 \leq y \leq d$ .

In the lower, half-plane the solution is obtained symmetrically.

First, we consider the solution of the system of Eqs (7) – (9) in the shear flow region. Here is  $\frac{dv}{dy} \neq 0$  everywhere except extreme points. Therefore, from Eq. (9) it follows

$$\tau_{xx} = \tau_{yy} \quad (10)$$

and rheological equation (8) is written as

$$\tau_{xy} = -\tau_0 + \mu \frac{dv}{dy} - \lambda \left(\frac{dv}{dy}\right)^2 \quad (11)$$

from which we conclude that  $\tau_{xy}$  is a function of only the variable  $y$ . This allows Eq. (7) to be written in a simpler form analogously to the work (Gnoevoj et al. 2004):

$$\frac{\partial \tau_{xx}}{\partial x} + \frac{d\tau_{xy}}{dy} = 0, \quad \frac{\partial \tau_{yy}}{\partial y} = 0 \quad (12)$$

On the basis of the second equality (12) and Eq. (10) we conclude

$$-\frac{d\tau_{xx}}{dx} = \frac{d\tau_{xy}}{dy} = \kappa \quad (13)$$

where  $\kappa$  is constant.

The integration of Eqs (13) and the use of boundary conditions for pressure leads to expressions for stresses (stress is opposite to pressure) analogously to the work (Mo et al. 2015):

$$\tau_{xy} = -\frac{\Delta p}{l} y + C, \quad \tau_{xx} = \tau_{yy} = -p_1 + \frac{\Delta p}{l} x,$$

where  $\Delta p = p_1 - p_2$ ,  $C$  is integration constant determined from symmetry of the problem relative to the plane  $y = 0$ . Due to this symmetry the linear function  $\tau_{xy}(y)$  is to be odd:  $\tau_{xy}(y) = -\tau_{xy}(-y)$  where we get  $C=0$ . Thus we have finally for stresses:

$$\tau_{xy} = -\frac{\Delta p}{l} y, \quad \tau_{xx} = \tau_{yy} = -p_1 + \frac{\Delta p}{l} x \quad (14)$$

Substitute in the rheological equation (11) the dependence (14) for  $\tau_{xy}$ :

$$-\frac{\Delta p}{l} y = -\tau_0 + \mu \frac{dv}{dy} - \lambda \left(\frac{dv}{dy}\right)^2 \quad (15)$$

From here we explicitly express the velocity gradient  $\frac{dv}{dy}$ :

$$\frac{dv}{dy} = \frac{\mu}{2\lambda} - \sqrt{\frac{\Delta p}{l\lambda} (y + a)} \quad (16)$$

where

$$a = \frac{l}{\Delta p} \left( \frac{\mu^2}{4\lambda} - \tau_0 \right) \tag{17}$$

Elementary integrating Eq. (16) with applying the boundary condition  $v = 0$  at  $y = d$  leads to an expression for the shear flow rate of quasi Bingham medium:

$$v = \frac{\mu}{2\lambda} (y - d) + \frac{2}{3} \sqrt{\frac{\Delta p}{\lambda l}} [(d + a)^{3/2} - (y + a)^{3/2}] \tag{18}$$

In this case, the effective dynamic viscosity is determined by the equality:

$$\mu_{eff} = \mu - \lambda \frac{dv}{dx} = \frac{\mu}{2} + \sqrt{\frac{\lambda \Delta p}{l}} (y + a) \tag{19}$$

It is seen from this expression that the effective viscosity increase with increasing a variable  $y$  which in turn means that this viscosity increases as the shear rate gradient increases. The latter indicates the manifestation of dilatant properties of quasi Bingham medium.

Let us consider the area of plastic flow in which the flow velocity is maximum in comparison with the flow velocity in the shear area. In this connection we find a maximum point  $y_1$  of the function (18) using a necessary condition of extremum of this function  $\frac{dv}{dy} = 0$ :

$$y_1 = \frac{l}{\Delta p} \tau_0 \tag{20}$$

The point  $y_1$  determines the boundary between areas of shear and plastic flows. It is easy to make sure that the sufficient maximum condition at the point  $y_1$  is also satisfied:

$$\frac{d^2v}{dy^2} \Big|_{y=y_1} = -\frac{\Delta p}{\mu l} \sqrt{\lambda} < 0.$$

Substituting Eq. (20) into Eq. (18) we find a maximum value of shear velocity  $v_{max}^s$  which is a constant velocity of plastic flow  $v^p$ :

$$v_{max}^s = v^p = \frac{\mu}{2\lambda} (y_1 - d) + \frac{2}{3} \sqrt{\frac{\Delta p}{\lambda l}} [(d + a)^{3/2} - (y_1 + a)^{3/2}] \tag{21a}$$

or finally

$$v^p = \frac{l}{\Delta p} \left\{ -\frac{\mu}{2\lambda} \left( \frac{\Delta p}{l} d + \frac{\mu^2}{6\lambda} - \tau_0 \right) + \frac{2}{3\sqrt{\lambda}} \left( \frac{\Delta p}{l} d + \frac{\mu^2}{4\lambda} - \tau_0 \right)^{3/2} \right\} \tag{21b}$$

Simple calculations show that at the boundary of the plastic flow, that is, at the point  $y_1$ :  $\mu_{eff}(y_1) = \mu$ . It is mean that the plastic flow has the viscosity not equal zero.

Since the velocity of the plastic flow (21a), (21b) is constant its derivative is zero at any point in this region:

$$\frac{dv^p}{dy} = 0.$$

Therefore, the rheological equation (8) takes the form:

$$\frac{1}{2} \sqrt{(\tau_{xx} - \tau_{yy})^2 + 4\tau_{xy}^2} = -\tau_0 \tag{22}$$

This equation is similar to the one considered in (Gnoevoj et al. 2004). Therefore, a further description of the plastic flow is similar to the description given in (Mo et al. 2015) in which the following expressions of stresses are obtained:

$$\begin{aligned} \tau_{xy}^p(y) &= -\frac{\Delta p}{l} y, \quad \tau_{yy}^p = -p_1 + \frac{\Delta p}{l} x, \\ \tau_{xx}^p &= \tau_{yy}^p - 2 \sqrt{\tau_0^2 - \tau_{xy}^{p2}} \end{aligned} \tag{23}$$

From the comparison of Eqs (14) and (23), it is seen that at the boundary between the areas of shear and plastic flow the stresses coincide:

$$\tau_{xy}^s = \tau_{xy}^p, \quad \tau_{yy}^s = \tau_{yy}^p.$$

We calculate the flow rate  $W$  of a quasi Bingham medium flowing in a channel bounded by parallel planes:

$$W = \int_{-d}^d v(y) dy = 2 \left( \int_{y_1}^d v^s(y) dy + \int_0^{y_1} v^p dy \right),$$

using Eqs (18), (21). Calculations lead to the result:

$$\begin{aligned} W &= \left( \frac{l}{\Delta p} \right)^2 \left\{ \frac{\mu}{2\lambda} \left[ (\tau_0 - \frac{\mu^2}{6\lambda})^2 + \frac{\mu^4}{180\lambda^2} \right. \right. \\ &\quad \left. \left. - \left( \frac{\Delta p}{l} d \right)^2 \right] + \frac{4}{5\sqrt{\lambda}} \left( \frac{\Delta p}{l} d + \frac{\mu^2}{4\lambda} - \tau_0 \right)^{3/2} \cdot \left( \frac{\Delta p}{l} d - \frac{2}{3} \left( \frac{\mu^2}{4\lambda} - \tau_0 \right) \right) \right\} \end{aligned} \tag{24}$$

Note that in the special case of absence of dynamic viscosity ( $\mu=0$ ) we obtain the Herschel - Bulkley model successfully describing rheology of weighted drilling fluids (Bulatov 2016) with the rheological law

$$\tau = \tau_0 - \lambda(\dot{\gamma})^2.$$

Within the last, we have from Eqs (18), (21b), (24) respectively:

$$v^s = \frac{2}{3} \sqrt{\frac{\Delta p}{\lambda l}} \left[ \left( d + \frac{l\tau_0}{4\Delta p\lambda} \right)^{3/2} - \left( y + \frac{l\tau_0}{4\Delta p\lambda} \right)^{3/2} \right],$$

$$v^p = \frac{2}{3} \sqrt{\frac{\Delta p}{\lambda l}} \left( d - \frac{l\tau_0}{\Delta p} \right)^{3/2},$$

$$W = \left( \frac{l}{\Delta p} \right)^2 \left\{ W = \frac{4}{5} \sqrt{\frac{\Delta p}{\lambda l}} \left( d - \frac{l\tau_0}{\Delta p} \right)^{3/2} \cdot \left( d + \frac{2}{3} \frac{l\tau_0}{\Delta p} \right) \right\}$$

## CONCLUSION

Summarizing the above, it should be noted that experimentally obtained rheograms of slag suspensions based on slag samples of Novokuznetsk, Mariupol, Chelyabinsk metallurgical plants show that their rheological properties differ from those of Bingham media. Precisely, they detect a nonlinear dependence of the shear stress on the flow velocity gradient. Based on these experimental results, it is proposed a new model of description of the above viscoplastic suspensions with the nonlinear rheological law taking into account the manifestation of dilatant properties of the considered suspensions called as quasi Bingham media. As an example, the stationary flow of the quasi-Bingham medium is considered in the channel bounded by parallel planes at a finite pressure difference. It is obtained exact solution of the system of equations describing the considered flow.

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